## Contents

Preface v

1 Definitions and basic results 1
  1.1 Convex functions 1
  1.2 Superquadratic and subquadratic functions 7
  1.3 Operator convex functions 8
  1.4 Fractional integrals and fractional derivatives 9

2 Some new Hardy-type inequalities with general kernels 15
  2.1 Preliminaries 15
  2.2 The main results 20
  2.3 Remarks and examples 25

3 On an inequality of G. H. Hardy 31
  3.1 New inequalities involving fractional integrals and derivatives 43
  3.2 Improvements of an inequality of G. H. Hardy 48

4 Some new refined Hardy-type inequalities with kernels 55
  4.1 New general refined Hardy-type inequalities with kernels 55
  4.2 One-dimensional refined Hardy-Hilbert-type inequalities 71
  4.3 Refined Godunova-type inequalities 74
  4.4 Refinements of an inequality of G. H. Hardy 77

5 Refinements of Hardy-type inequalities for the case $0 < p \leq q < \infty$ 89
  5.1 A new class of general Hardy-type inequalities with kernels 89
    5.1.1 Further results involving fractional integrals and derivatives 91
  5.2 Refined Hardy-type inequalities with kernels 103
  5.3 Generalized one-dimensional Hardy’s and Pólya-Knopp’s inequality 109
  5.4 Generalized one-dimensional Hardy-Hilbert’s inequality 115
  5.5 General Godunova-type inequalities 120
  5.6 Generalized G. H. Hardy-type inequality 121
    5.6.1 G. H. Hardy-type inequalities for fractional integrals 122
    5.6.2 G. H. Hardy-type inequalities for fractional derivatives 127
12 The Boas functional and its properties

12.1 The Boas functional and exponential convexity .......................... 265
12.2 Mean value theorems related to the Boas functional ..................... 269
12.3 Cauchy-type means generated by the Boas functional ................... 271