

Contents

Preface	v
1 Zipf-Mandelbrot law, properties and its generalizations	1
(<i>J. Jakšetić, Đ. Pečarić and J. Pečarić</i>)	
1.1 Some classical inequalities and Zipf-Mandelbrot law	1
1.1.1 Introduction	1
1.1.2 Zipf law estimations	3
1.1.3 Zipf law and Goldman inequality	6
1.1.4 Further bounds via Lyapunov and Hölder inequality	7
1.2 Analytical properties of Zipf-Mandelbrot law and Hurwitz ζ -function	9
1.2.1 Monotonicity properties	10
1.2.2 Log-convexity and exponential convexity	12
1.2.3 Log subadditivity	14
1.2.4 Gini means and further monotonicity	15
1.3 Hybrid Zipf-Mandelbrot law	17
1.3.1 Shannon entropy and Zipf-Mandelbrot law	17
1.3.2 Hybrid Zipf-Mandelbrot law	18
1.3.3 Properties of the hybrid Zipf-Mandelbrot law	21
1.3.4 Hybrid means	25
2 On Zipf-Mandelbrot entropy	29
(<i>M. A. Khan, Đ. Pečarić, J. Pečarić and Z. M. Al-Sahwi</i>)	
2.1 Introduction	29
2.2 Preliminary results on Shannon inequality	30
2.3 Inequalities for Zipf-Mandelbrot entropy	35
3 Approximating f-divergence via Hermite interpolating polynomial	45
(<i>Đ. Pečarić, J. Pečarić and A. Vukelić</i>)	
3.1 Introduction	45
3.2 Cauchy's error representation and inequalities for f -divergence	47
3.3 Hermite-Hadamard type inequalities	51
3.4 Inequalities by using the Zipf-Mandelbrot law	56

4 Zipf-Mandelbrot law and superadditivity of the Jensen functional	
(<i>N. Lovričević, Đ. Pečarić and J. Pečarić</i>)	61
4.1 Introduction and preliminaries	62
4.2 f - divergences and superadditivity of the Jensen functional	65
4.3 Zipf-Mandelbrot law in f - divergences	73
5 Inequalities for Shannon and Zipf-Mandelbrot entropies by using Jensen's type inequalities	
(<i>M. A. Khan, Đ. Pečarić and J. Pečarić</i>)	89
5.1 Introduction	90
5.2 Discrete Jensen's type inequalities	92
5.3 Integral Jensen's type inequalities	94
5.4 Bounds for Shannon entropy	96
5.5 Results for Csiszar divergence	98
5.6 Bounds for Zipf-Mandelbrot entropy	103
5.7 Results for hybrid Zipf-Mandelbrot entropy	116
6 Combinatorial improvements of Zipf-Mandelbrot Laws via interpolations	
(<i>K. A. Khan, T. Niaz, Đ. Pečarić and J. Pečarić</i>)	129
6.1 Introduction	130
6.1.1 Inequalities for Csiszár divergence	133
6.1.2 Inequalities for Shannon Entropy	135
6.1.3 Inequalities for Rényi Divergence and Entropy	136
6.2 Inequalities by Using Zipf-Mandelbrot Law	142
6.3 Generalization of refinement of Jensen's, Rényi and Shannon type inequalities via Green Function	148
6.4 Generalization of refinement of Jensen's, Rényi and Shannon type inequalities via Montgomery identity	149
6.5 Generalization of refinement of Jensen's, Rényi and Shannon type inequalities via Lidstone Polynomial	152
6.6 Generalization of refinement of Jensen's, Rényi and Shannon type inequalities via Taylor Polynomial	155
6.7 Generalization of refinement of Jensen's, Rényi and Shannon type inequalities via Hermite Interpolating Polynomial	157
7 Improvements of the inequalities for the f-divergence functional with applications to the Zipf-Mandelbrot law	
(<i>S. I. Butt, L. Horváth, Đ. Pečarić and J. Pečarić</i>)	167
7.1 Introduction	168
7.2 Estimations of f - and Rényi divergences	171
7.2.1 Inequalities for Csiszár divergence and Shannon entropy	171
7.2.2 Inequalities for Rényi divergence and entropy	174
7.2.3 Inequalities by using the Zipf-Mandelbrot law	180
7.3 Cyclic improvements of inequalities for entropy of Zipf-Mandelbrot law via Hermite interpolating polynomial	181

7.3.1	Extensions of cyclic refinements Of Jensen's inequality via Hermite interpolating polynomial	182
7.3.2	Cyclic improvements Of inequalities for entropy of Zipf-Mandelbrot law via Hermite polynomial	184
7.4	A refinement and an exact equality condition for the basic inequality of f -divergences	189
7.4.1	Construction of the equality conditions and related results of classical integral Jensen's inequality	189
7.4.2	Refinements of basic inequality in f -divergences and related results	196
8	Inequalities of the Jensen and Edmundson-Lah-Ribarič type for Zipf-Mandelbrot law	
(<i>R. Mikić, Đ. Pečarić, J. Pečarić and J. Perić</i>)		205
8.1	Introduction	206
8.2	Preliminaries	208
8.3	Inequalities for generalized f -divergence	216
8.4	Applications to Zipf-Mandelbrot law	236
8.5	Further generalization of Edmundson-Lah-Ribarič inequality for Zipf-Mandelbrot law	246
9	On Sherman's inequality with applications in information theory	
(<i>A. Barbir, S. Ivelić Bradanović, Đ. Pečarić and J. Pečarić</i>)		257
9.1	Introduction and preliminaries	257
9.2	The concept of Csiszár f -divergence functional	261
9.3	The Zipf-Mandelbrot entropy	264
9.4	Converse to Sherman's Inequality	265
9.5	Converses including some entropies and divergences	268
9.6	Converses including Zipf-Mandelbrot entropy	273
9.7	Bounds for Sherman's difference	275
9.8	Bounds for Csiszár f -divergence functional	277
9.9	Bounds for some entropies and divergences	279
9.10	Bounds for the Zipf-Mandelbrot entropy	282
10	Jensen-type inequalities for generalized f-divergences and Zipf-Mandelbrot law	
(<i>Đ. Pečarić, J. Pečarić, D. Pokaz and M. Rodić</i>)		287
10.1	Introduction	287
10.2	Preliminary results	288
10.3	Jensen's type inequalities for the generalized Csiszár f -divergence	290
10.4	Inequalities for different types of generalized f -divergences	293
10.5	The mappings of H and F	299
10.6	Applications to Zipf-Mandelbrot law	302

11 On Shannon and Zipf-Mandelbrot entropies and related results	
(<i>S. Khalid, Đ. Pečarić and J. Pečarić</i>)	313
11.1 Introduction and Preliminaries	313
11.2 On Zipf-Mandelbrot entropy and 3-convex functions	317
11.2.1 Inequalities related to Zipf-Mandelbrot entropy	317
11.2.2 Linear functionals and mean value theorems	320
11.2.3 n -exponential convexity and log-convexity	322
11.3 Shannon and Zipf-Mandelbrot entropies and related results	328
11.3.1 Inequalities related to Shannon entropy	328
11.3.2 Inequalities related to Zipf-Mandelbrot entropy	331
11.3.3 Linear functionals and mean value theorems	332
11.3.4 n -exponential convexity and log-convexity	333
List of Authors	341